

Some Theoretical Developments in SUSY

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Abstract

I review theoretical developments of the last year or so in nonperturbative supersymmetry.

Topics:

- Metastable vacua at strong coupling;
- Non-Abelian flux tubes, confined monopoles
- One step beyond Seiberg's duality
- Planar equivalence
- B theory (multileg/multiloop amplitudes)
- $\mathcal{N} = (2, 0)$ sigma model (Heterotic flux tubes of Edalati-Tong)

1 Metastable supersymmetry-breaking vacua in SQCD

I will start from a topic which lies half-way between purely theoretical aspects of supersymmetry (SUSY) and SUSY-based model-building. As well known, it is not easy to break supersymmetry dynamically in such a way that no unwanted phenomenological consequences occur. Usually one has to deal with contrived schemes which are not elegant, to put it mildly. A few mechanisms of dynamical SUSY breaking were discovered in 1980's and '90s (for a review see [1]). Approximately at the same time people realized that some generalized Wess-Zumino models can contain, in addition to supersymmetric vacua, local minima with positive energy density. If the barrier between the latter and supersymmetric vacua is high enough they represent long-lived metastable vacua in which supersymmetry is spontaneously broken [2]. The models that were built on such vacua in 1990s tended to be rather awkward.

In 2006 Intriligator, Seiberg and Shih showed [3] that metastable dynamical SUSY breaking is much more generic and much simpler than was previously thought. They considered $\mathcal{N} = 1$ $SU(N_c)$ SQCD with $N_c + N$ flavors ($N < N_c/2$) as the starting microscopic theory. As usual, each quark flavor is described by two chiral superfields, $Q^{k,A}$ and \tilde{Q}_{Ak} where k is the color index ($k = 1, 2, \dots, N_c$) and A is the flavor index ($A = 1, 2, \dots, N_c + N$). All quark flavors were endowed by a common mass term m assumed to be much smaller than the dynamical scale parameter Λ . This theory, to be referred to as the electric theory, is strongly coupled and its dynamics, beyond some general aspects (such as the number of SUSY vacua), is not amenable to exhaustive analysis.

The vacuum structure of this model can be studied through its magnetic Seiberg's dual which is in the infrared free regime. The quark mass term is converted into a crucial term in the superpotential of the magnetic (macroscopic) theory

$$\mathcal{W} = \mathcal{W}_{\text{tree}} + \mathcal{W}_{\text{anom}} , \quad (1)$$

where

$$\mathcal{W}_{\text{tree}} = \tilde{h}_{A,k} h^{kB} M_B^A - \mu^2 M_A^A , \quad (2)$$

$\tilde{h}_{A,k}$ and h^{kB} denote dual quarks, $k = 1, 2, \dots, N$, and $\mu \propto \sqrt{m}$.

The tree-level superpotential $\mathcal{W}_{\text{tree}}$ yields metastable vacua. The anomalous part of the superpotential, $\mathcal{W}_{\text{anom}} \propto \text{Tr} W^2$, is responsible for N_c vacua that restore supersymmetry. As long as $m \ll \Lambda$ supersymmetric vacua lie far away from the metastable vacua and are separated by a huge barrier. The lifetime of the metastable vacua can be made longer than the Universe lifetime. The same condition $m \ll \Lambda$ allows one to control the incalculable Kähler potential corrections.

When all quark mass terms are equal, the electric theory possesses a global vector-like $SU(N_c + N) \times U(1)_B$ symmetry. This is a flavor symmetry which is spontaneously broken down to $S(U(N) \times U(N_c))$ in the metastable vacua. Thus, the latter form a compact moduli space of metastable vacua

$$\mathcal{V} = \frac{U(N_c + N)}{S(U(N) \times U(N_c))}, \quad (3)$$

to be compared with N_c isolated stable vacua.

The Intriligator-Seiberg-Shih (ISS) finding opens for investigation a large class of (hopefully simple) phenomenologically relevant models with dynamical SUSY breaking. Contentious issues such as large flavor symmetries and the absence of an R symmetry, a usual “bottle neck” in model-building, may find relatively easy solutions [4]. An aspect of model-building where the ISS finding may prove promising is constructing a “direct mediation” model, i.e. a model where the messengers are an integral part of the SUSY-breaking sector. Since SQCD and its cousins naturally come with large global flavor symmetries, one could imagine gauging such a symmetry and identifying it with the Standard Model. Such a model might have different phenomenology compared with ordinary gauge mediation.

The aspect which is of most importance to me in this talk is purely theoretical. The ISS work raises the question of where small deformations of Seiberg’s duality can lead us. We will see later (Sect. 3) that they will lead us pretty far — to non-Abelian strings and confined monopoles.

2 Non-Abelian flux tubes and confined monopoles

Seiberg and Witten [5] presented the first ever demonstration of the dual Meissner effect in non-Abelian theory, a celebrated analytic proof of linear confinement, which caused much excitement in the community.

It took people three years to realize [6] that the flux tubes in the Seiberg–Witten solution are not those we would like to have in QCD. Hanany, Strassler and Zaffaroni who analyzed in 1997 the chromoelectric flux tubes in the Seiberg–Witten solution showed that these flux tubes are essentially Abelian (of the Abrikosov–Nielsen–Olesen type) so that the hadrons they would create would have nothing to do with those in QCD. The hadronic spectrum would be significantly richer. And, say, in the $SU(3)$ case, three flux tubes in the Seiberg–Witten solution would not annihilate into nothing, as they should in QCD ...

Ever since searches for genuinely non-Abelian flux tubes and non-Abelian monopoles continued, with a decisive breakthrough in 2003-04. By that time the program of finding field-theory analogs of all basic constructions of string/D-brane theory

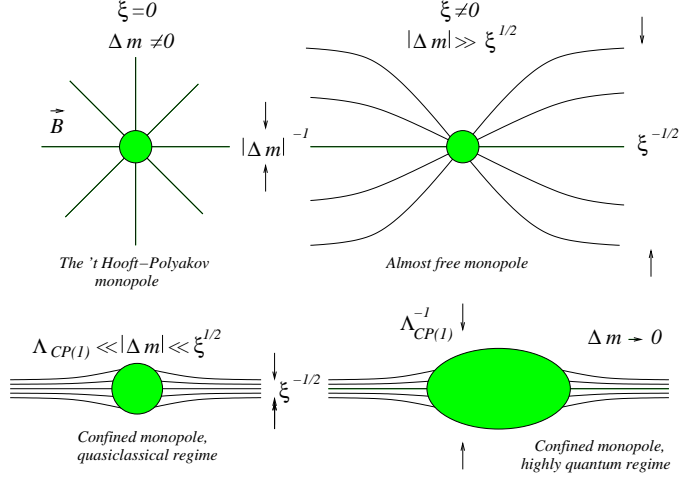


Figure 1: Various regimes for monopoles and strings.

was in full swing.¹ BPS domain walls, analogs of D branes, had been identified in supersymmetric Yang–Mills theory. It had been demonstrated that such walls support gauge fields localized on them. and BPS saturated string-wall junctions had been constructed [8]. And yet, non-Abelian flux tubes, the basic element of the non-Abelian Meissner effect, remained elusive.

2.1 Non-Abelian flux tubes

They were first found [9, 10] in U(2) super-Yang–Mills theories with extended supersymmetry, $\mathcal{N} = 2$, and two matter hypermultiplets. If one introduces a non-vanishing Fayet–Iliopoulos parameter ξ the theory develops isolated quark vacua, in which the gauge symmetry is fully Higgsed, and all elementary excitations are massive. In the general case, two matter mass terms allowed by $\mathcal{N} = 2$ are unequal, $m_1 \neq m_2$. There are free parameters whose interplay determines dynamics of the theory: the Fayet–Iliopoulos parameter ξ , the mass difference Δm and a dynamical scale parameter Λ , an analog of the QCD scale Λ_{QCD} . Extended supersymmetry guarantees that some crucial dependences are holomorphic, and there is no phase transition.

The number of colors can be arbitrary. The benchmark model supporting non-Abelian flux tubes has the gauge group $\text{SU}(N) \times \text{U}(1)$ and N flavors. The $\mathcal{N} = 2$ vector multiplet consists of the U(1) gauge field A_μ and the $\text{SU}(N)$ gauge field A_μ^a ,

¹This program started from the discovery of the BPS domain walls in $\mathcal{N} = 1$ supersymmetric gluodynamics [7].

(here $a = 1, \dots, N^2 - 1$), and their Weyl fermion superpartners (λ^1, λ^2) and $(\lambda^{1a}, \lambda^{2a})$, plus complex scalar fields a , and a^a . The latter are in the adjoint representation of $SU(N)$. In this sector the global $SU(2)_R$ symmetry inherent to $\mathcal{N} = 2$ models manifests itself through rotations $\lambda^1 \leftrightarrow \lambda^2$.

The quark multiplets of the $SU(N) \times U(1)$ theory consist of the complex scalar fields q^{kA} and \tilde{q}_{Ak} (squarks) and the Weyl fermions ψ^{kA} and $\tilde{\psi}_{Ak}$, all in the fundamental representation of the $SU(N)$ gauge group ($k = 1, \dots, N$ is the color index while A is the flavor index, $A = 1, \dots, N$). The scalars q^{kA} and \tilde{q}^{kA} form a doublet under the action of the global $SU(2)_R$ group. Quarks and squarks have a $U(1)$ charge too. A $U(1)$ Fayet-Iliopoulos (FI) term is introduced, with the FI parameter ξ .

Both gauge and flavor symmetries of the model are broken by the condensation of scalar fields. A global diagonal combination of color and flavor groups, $SU(N)_{C+F}$, survives the breaking (the subscript $C + F$ means a combination of global color and flavor groups). While $SU(N)_{C+F}$ is the symmetry of the vacuum, the flux tube solutions break it spontaneously. This gives rise to orientational moduli. Topological stability of the non-Abelian strings in this model is due to the fact that $\pi_1(U(N)/Z_N) = Z_N \times Z$.

2.2 Confined monopoles

As various parameters vary, this theory evolves in a very graphic way, see Fig. 1. At $\xi = 0$ but $\Delta m \neq 0$ (and $\Delta m \gg \Lambda$) it presents a very clear-cut example of a model with the standard 't Hooft–Polyakov monopole. The monopole is free to fly — the flux tubes are not yet formed.

Switching on $\xi \neq 0$ traps the magnetic fields inside the flux tubes, which are weak as long as $\xi \ll \Delta m$. The flux tubes change the shape of the monopole far away from its core, leaving the core essentially intact. Orientation of the chromomagnetic field inside the flux tube is essentially fixed. The flux tubes are Abelian.

With $|\Delta m|$ decreasing, fluctuations in the orientation of the chromomagnetic field inside the flux tubes grow. Simultaneously, the monopole which no longer resembles the 't Hooft–Polyakov monopole, is seen as the string junction. It acquires a life of its own.

Finally, in the limit $\Delta m \rightarrow 0$ the transformation is complete. A global $SU(2)$ symmetry restores in the bulk. Orientational moduli develop on the string worldsheet making it truly non-Abelian. The string worldsheet theory is $CP(1)$ ($CP(N-1)$ for generic values of N). Two-dimensional $CP(N-1)$ models with four supercharges are asymptotically free. They have N distinct vacuum states. The non-Abelian flux tubes have degenerate tensions $T_{st} = 2\pi\xi$. The ANO string tension is N times larger.

Each vacuum state of the worldsheet $CP(N-1)$ theory presents a distinct string from the standpoint of the bulk theory. There are N species of such strings. All

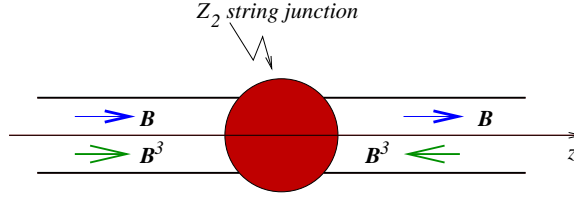


Figure 2: Z_2 string junction.

have the same tension. Hence, two different strings form a stable junction. Figure 2 shows this junction in the limit

$$\Lambda_{\text{CP}(1)} \ll |\Delta m| \ll \sqrt{\xi} \quad (4)$$

corresponding to the lower left corner of Fig. 1. The magnetic fluxes of the U(1) and SU(2) gauge groups are oriented along the z axis. In the limit (4) the SU(2) flux is oriented along the third axis in the internal space. However, as $|\Delta m|$ decreases, fluctuations of B_z^a in the internal space grow, and at $\Delta m \rightarrow 0$ it has no particular orientation in SU(2) (the lower right corner of Fig. 1). In the language of the worldsheet theory this phenomenon is due to restoration of the O(3) symmetry in the quantum vacuum of the CP(1) model.

The junctions of degenerate strings present what remains of the monopoles in this highly quantum regime [11, 12]. It is remarkable that, despite the fact we are deep inside the highly quantum regime, holomorphy allows one to exactly calculate the mass of these monopoles. This mass is given by the expectation value of the kink central charge in the worldsheet CP($N - 1$) model (including the anomaly term).

What remains to be done? The most recent investigations zero in on $\mathcal{N} = 1$ theories, which are much closer relatives of QCD than $\mathcal{N} = 2$. I have time to say just a few words on the so-called M model suggested recently [13] which seems quite promising.

2.3 M model

The unwanted feature of $\mathcal{N} = 2$ theory, making it less similar to QCD, is the presence of the adjoint scalar field. One can get rid of it making it heavy. To this end we must endow the adjoint superfield by a mass term. Supersymmetry of the model becomes $\mathcal{N} = 1$. Moreover, to avoid massless modes in the bulk theory (in the limit of very heavy adjoint fields) we must introduce a “meson” superfield M_B^A analogous to that emerging in the magnetic Seiberg dual, see Sect. 1, with an appropriately superpotential. After the adjoint field is eliminated the theory has no ’t Hooft–Polyakov monopoles in the quasiclassical limit. Nevertheless, a non-Abelian

Meissner effect does take place: condensation of color charges (squarks) gives rise to non-Abelian flux tubes and confined monopoles. The very fact of their existence in $\mathcal{N} = 1$ supersymmetric QCD without adjoint scalars was not known previously. The analysis presented in Ref. [13] is analytic and is based on the fact that the $\mathcal{N} = 1$ theory under consideration can be obtained starting from $\mathcal{N} = 2$ SQCD in which the 't Hooft–Polyakov monopoles do exist, through a certain limiting procedure allowing one to track the status of these monopoles at various stages (analogous to the one described above and summarized in Fig. 1).

The M model shares many features with the ISS magnetic theory. I will return to this fact later. Now I would like to note that non-BPS flux tubes in the metastable vacua of the $\text{SO}(N_c)$ ISS theory with $N_c + N - 4$ flavors were found by Eto et al. [14]. In a parallel consideration, they made one extra step on the way from the $\text{SU}(N_c)$ ISS magnetic theory towards the M model. They gauged the baryon $\text{U}(1)$. The $\text{U}(N)$ magnetic theory obtained in this way supports flux tubes in the metastable vacua [14].

The M model can be regarded as the first cousin of QCD since the adjoint fields typical of $\mathcal{N} = 2$ are eliminated in this theory. Even though supersymmetry is considerably weakened, the overall qualitative picture survives. This is probably one of the most important findings at the current stage.

Can a dual of the M model be identified? If yes, this would be equivalent to the demonstration of the non-Abelian dual Meissner effect in $\mathcal{N} = 1$.

3 Dualizing (almost) M model

I started my talk from ISS who slightly mass-deformed SQCD. This small deformation led to drastic consequences in the infrared-free magnetic dual theory: emergence of a non-supersymmetric metastable vacuum.

It turns out that further quite mild deformations of this “electric” theory result in a dual “magnetic” theory which is very close to the M model discussed above. It preserves all salient features of the M model.

Shifman and Yung considered [15] $\mathcal{N} = 1$ SQCD with the gauge group $\text{U}(N_c)$ and $N_c + N$ quark flavors ($N < N_c/2$). The $\text{U}(1)$ gauge factor gauging baryon charge is the first (but not last) distinction from ISS.

The next distinction is that we keep N_c flavors massless; the corresponding squark fields develop (small) vacuum expectation values (VEVs) on the Higgs branch. Extra N flavors are endowed with a mass term m_q which is also small compared to Λ_Q , so that all fields are dynamical (none can be integrated out).

Within the framework of this deformation of Seiberg’s procedure, on the other side of duality, the IR free regime is deformed to give rise to a theory which has the gauge group $\text{U}(N)$, N_c massive dual-quark flavors plus N massless dual-quark

flavors. In addition to gauge interactions they are coupled to the meson field M_B^A through a superpotential. The massive flavors can be integrated out while the massless ones develop VEVs. The theory is fully Higgsed. The scales relevant to the electric and magnetic theories are indicated in Fig. 3.

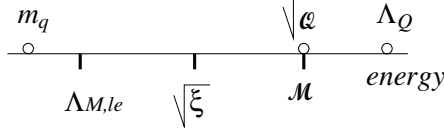


Figure 3: Scales of the electric (open points) and dual magnetic (dashes) theories.

This set-up leads one from a slightly deformed SQCD to the magnetic dual which is very close to the M model: it supports flux tubes (strings) at weak coupling and confines non-Abelian (dual) monopoles. The flux tubes are not BPS-saturated, unlike those of the M model. This is inessential.

A number of states in the magnetic theory are light in the sense that their mass tends to zero in the limit $m_q \rightarrow 0$. Via duality these light states are in one-to-one correspondence with the light states of the original electric theory. Thus, duality gets extended to include (in addition to massless moduli) a part of the spectrum which is light compared to the natural dynamical scale Λ_Q *but not massless*.

Extended duality allows one to analyze the magnetic theory at weak coupling and make a number of highly nontrivial predictions for the quark theory light sector which is at strong coupling. Non-Abelian monopoles must be important in Seiberg’s duality being related to “dual quarks.” We make one step further suggesting that the non-Abelian monopoles of the electric theory *are* the “dual quarks.” The dual quark fields condense providing (small) masses to all gauge bosons of the magnetic theory. The way the magnetic theory is Higgsed is very peculiar — it corresponds to baryon-operator dominated vacuum in the quark theory. Confined monopoles of the magnetic theory are to be interpreted as certain “constituent quarks” of the quark theory. Both form N -plets of the global unbroken $SU(N)$ symmetry which is present in the quark and monopole theories, on both sides of the extended duality.

In the quark theory color is screened since the theory is fully Higgsed. There are matter fields in the fundamental representation. Therefore long strings cannot exist. They are screened/ruptured immediately. On the dual side we do see strings, however. The scale of the string-induced confinement $\sqrt{\xi}$ is small in the original quark theory, much smaller than its dynamical scale, $\sqrt{\xi} \ll \Lambda_Q$.

This apparent puzzle can be resolved if we assume that a “secondary” gauge theory (or a “gauge cascade”) develops in the original quark theory. Assume that massless composite “ ρ mesons” whose size is $\sim \Lambda_Q^{-1}$ are formed in the quark theory which interact with each other via a “secondary” gauge theory whose scale parameter is $\sqrt{\xi}$. At distances $\sim 1/\sqrt{\xi}$ the above “ ρ mesons” must be viewed as massless

gluons. It is conceivable that they are coupled to massless “secondary” quarks which, in addition to their gauge coupling to “ ρ mesons”, have nontrivial quantum numbers with respect to the global $SU(N)$. With respect to the original quark theory the “secondary” quarks are colorless (“bleached”) bound states which include the original quarks at their core. Their sizes are proportional to $\sim \Lambda_Q^{-1}$ and, hence, they are pointlike on the scale of $\sim 1/\sqrt{\xi}$, much in the same way as “ ρ mesons”-gluons.

I think this is a very interesting interpretational issue which calls for further investigation. On the other hand, some people may want to avoid it. To this end one can follow the road which was suggested recently by Eto et al. [16]. Conceptually their strategy is very similar to that of their predecessors (Ref. [15]), with one crucial exception. Eto et al. base their electric theory on $SO(N_c)$ rather than $U(N_c)$. Then they endow some quarks (chiral superfields in the vector representation of $SO(N_c)$) with a mass terms, keeping other flavors massless. This eliminates metastable ISS vacua. The magnetic theory supports non-Abelian strings due to the fact that $\pi_1(SO(N)) = Z_2$. At the same time, spinor probe charges in the electric theory cannot be screened.

4 Chirally symmetric quasivacuum in supersymmetric gluodynamics?

Since mid-1980s it is known that strong- and weak-coupling calculations of the gluino condensate in supersymmetric gluodynamics do not match. For $SU(2)$ gauge theory the mismatch is $5/4$ (for a review see Ref. [1], Sects. 4.2, 4.3 and 7).

To explain the puzzle Kovner and Shifman suggested [17] that an extra chirally symmetric vacuum, with the vanishing gluino condensate, exists. This conclusion was also supported by the Veneziano-Yankielowicz effective Lagrangian [18] with the simplest kinetic term. However, later Cachazo et al. proved [19] that supersymmetric chirally symmetric vacuum is impossible.

The chirally symmetric vacuum may be revived in a new incarnation, of an unstable (non-supersymmetric) minimum. This directly follows from ISS. In their set-up the metastable vacuum is well defined as a manifold of critical points of $\mathcal{W}_{\text{tree}}$. In these vacua the “dynamically generated part” of the superpotential

$$\mathcal{W}_{\text{dyn}} \propto (\det M)^{1/N} = 0, \quad (5)$$

cf. Eqs. (1) and (2), since $M = 0$ in the supersymmetry breaking ISS solution.

The dynamically generated part of the superpotential is in fact a low-energy matrix element of the operator $\text{Tr} \lambda^2$, as it follows from the anomaly relation

$$\mathcal{W} = \mathcal{W}_{\text{tree}} + \frac{N_c}{16\pi^2} \text{Tr} W^2 \quad (6)$$

(for a derivation see e.g. Sect. 3.2 of the review [1]). Hence, in the metastable ISS vacua

$$\langle \text{Tr } \lambda \lambda \rangle_{\text{mv}} \rightarrow 0 \text{ as } M \rightarrow 0, \quad (7)$$

where the subscript mv stands for metastable vacuum. Making the quark mass m large we approach SUSY gluodynamics. Apparently, at $m \sim \Lambda$ the metastable vacuum becomes highly unstable. Nevertheless, it may well play the role required from the “Kovner–Shifman vacuum” in the strong-coupling instanton calculation of the gluino condensate.

Independent arguments in favor of the chirally symmetric unstable minimum were given by Douglas, Shelton and Torroba (see Sect. 5 of [20]).

5 Planar Equivalence

Planar equivalence is equivalence in the large- N limit of distinct QCD-like theories in their common sectors (see [21]). Most attention received equivalence between SUSY gluodynamics and its orientifold and Z_2 orbifold daughters. The Lagrangian of the parent theory is

$$\mathcal{L} = -\frac{1}{4g_P^2} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{i}{g_P^2} \lambda^{a\alpha} D_{\alpha\dot{\beta}} \bar{\lambda}^{a\dot{\beta}} \quad (8)$$

where $\lambda^{a\alpha}$ is the gluino (Weyl) field in the adjoint representation of $\text{SU}(N)$, and g_P^2 stands for the coupling constant in the parent theory. The orientifold daughter is obtained by replacing $\lambda^{a\alpha}$ by the *Dirac* spinor in the two-index (symmetric or antisymmetric) representation (to be referred to as orienti-S or orienti-AS). The gauge coupling stays intact. To obtain the Z_2 orbifold daughter (to be referred to as orbi) we must pass to the gauge group $\text{SU}(N/2) \times \text{SU}(N/2)$, replace $\lambda^{a\alpha}$ by a bifundamental Dirac spinor, and rescale the gauge coupling, $g_D^2 = 2g_P^2$.

5.1 Brief history

Genesis of planar equivalence can be traced to string theory. In 1998 Kachru and Silverstein studied [22] various orbifolds of R^6 within the AdS/CFT correspondence, of which I will speak later. Starting from $\mathcal{N} = 4$, they obtained distinct — but equivalent in the infinite- N limit — four-dimensional daughter gauge field theories with matter, with varying degree of supersymmetry, all with vanishing β functions.²

The next step was made by Bershadsky et al. [23]. These authors eventually abandoned AdS/CFT, and string methods at large. Analyzing gauge field theories *per se* they proved that an infinite set of amplitudes in the orbifold daughters of the

²This statement is slightly inaccurate; I do not want to dwell on subtleties.

parent $\mathcal{N} = 4$ theory in the large- N limit coincide with those of the parent theory, order by order in the gauge coupling. Thus, explicitly different theories have the same planar limit, at least perturbatively.

After a few years of relative oblivion, interest in the issue of planar equivalence was revived by Strassler [24]. He shifted the emphasis away from the search for supersymmetric daughters, towards engineering QCD-like daughters. Strassler considered Z_N orbifolds. In 2003 an orientifold daughter of SUSY gluodynamics was suggested as a prime candidate for nonperturbative equivalence [25, 21]. At $N = 3$ this orientifold daughter identically reduces to one-flavor QCD! Thus, one-flavor QCD is planar-equivalent to SUSY gluodynamics. This remarkable circumstance allows one to copy results of these theories from one to another. For instance, color confinement of one-flavor QCD to supersymmetric Yang–Mills, and the exact gluino condensate in the opposite direction. This is how the quark condensate was calculated, for the first time analytically, in one-flavor QCD [26].

5.2 Recent Developments

Kovtun, Ünsal and Yaffe formulated (and derived) [27, 28] the necessary and sufficient conditions for nonperturbative planar equivalence to be valid. This condition is nonbreaking of discrete symmetries: interchange Z_2 invariance for the Z_2 orbifold daughter, and C invariance for the orientifold daughter. Although at first glance it does not seem to be a hard problem to prove that spontaneous breaking of the discrete symmetries does not occur, in fact, this is a challenging problem which defies exhaustive solution so far.

The question of the discrete symmetry nonbreaking would be automatically solved if one could prove that the expansion in fermion loops (say, for the vacuum energy) is convergent in some sense [29].

To be more exact, let us give a mass term m to the fermions, and assume at first this mass term to be large, $m \gg \Lambda$. Then the N_f expansion is certainly convergent. The question is “is there a singularity, so that at small m the convergence is lost?” I believe that there is no such singularity. If so, both Z_2 orbi and orienti-S/AS are nonperturbatively equivalent to supersymmetric gluodynamics. Note that this statement does not refer to Z_N orbi with $N > 2$. In this case no mass term is possible in the orbi theory, it is chiral.

On what I base my belief? Consider supersymmetric gluodynamics with SUSY slightly broken by a small mass term of gluino. At $N = \infty$ the vacuum structure of this theory is exactly the same as the vacuum structure of pure Yang–Mills (the latter was derived by Witten [30], see also [31, 32]). Thus, I would say that the expansion in the number of fermion loops should work. This is of course not a mathematical theorem, but rather a physics argument.

Since for given number of fermion loops and given $m \neq 0$ each expansion term in

supersymmetric gluodynamics is exactly the same as the corresponding expansion term in Z_2 orbi and orienti-S/AS, the fermion loop expansions in all three theories must be convergent.

Since in pure gauge theory, with no fermions, the vacuum is unique [30], then so is the case for Z_2 orbi and orienti-S/AS at $m \neq 0$. The uniqueness of the vacuum state (for $\theta = 0$) implies the absence of the spontaneous breaking of the discrete symmetries in the above daughter theories.

If the statement is valid for small $m \neq 0$ extrapolation to $m = 0$ must be smooth since none of these theories has massless particles in the limit $m = 0$. They all have a mass gap $\sim \Lambda$.

The studies of planar equivalence reminded us of a remarkable fact: so far we have no rigorous proof of the absence of the spontaneous breaking of C invariance in QCD-like theories [28, 33]. For P invariance such proof exists [34], it is iron-clad, but essentially non-dynamical. Can we find a proof of C invariance? (Of course, in QCD *per se* this fact is well established empirically.)

5.3 Center-group symmetry and the limit $N \rightarrow \infty$

The planar equivalence between the parent and daughter theories described in the beginning of Sect. 5 holds not only on R_4 but in arbitrary geometry. Therefore, one can compare phase diagrams and, in particular, temperature dependences. This topic was open by Sannino [35], a thorough discussion was presented recently by Ünsal [36].

There is a famous Polyakov criterion regarding confinement/deconfinement in $SU(N)$ Yang–Mills theories. If one compactifies R_4 into $R_3 \times S_1$ and considers the Polyakov line along the compactified direction, its expectation value may or may not vanish. If it does not vanish, the Z_N symmetry — the center of the gauge $SU(N)$ group — is broken. On the other hand, if the Polyakov line vanishes the Z_N symmetry is unbroken. The former case corresponds to deconfinement, the latter to confinement.

Introducing quarks may bring in a problem with this criterion, since there is no center-group symmetry, generally speaking. This is in one-to-one correspondence with the fact that there are no genuine long strings in QCD. They break through quark-antiquark pair creation.

Here the $N = \infty$ limit helps. In $SU(N)$ Yang–Mills theory with quarks in the fundamental representation in the 't Hooft limit the fundamental quarks decouple, we are left with pure Yang–Mills which does have center group, and once it is broken, the theory is in the confinement phase. The Polyakov line is a good order parameter. The same is valid with regards to supersymmetric gluodynamics even at finite N . Gluinos do not decouple at large N , but they do not ruin the center-group symmetry.

However, consider, for instance, the AS orientifold daughter of supersymmetric

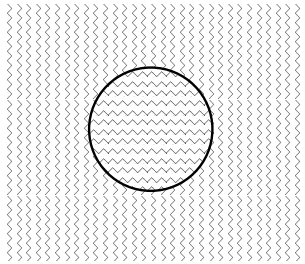


Figure 4: One fermion loop (i.e. $\log \det (i \not{\partial} + \not{A}^a T_{AS}^a)$) in the gluon field background (shown as shaded areas).

gluodynamics. At $N = \infty$ two-index antisymmetric fermions do *not* decouple. There is no apparent center-group symmetry in this theory, right? It is clear that the confinement/deconfinement criterion as the center-group breaking vs. nonbreaking is in trouble. Through planar equivalence we know that at $N = \infty$ the temperature behavior of this theory is exactly the same as in supersymmetric gluodynamics where the Polyakov criterion is perfectly applicable. Where is a way out?

What seems obvious is not always correct. I want to argue that the center-group symmetry which is not seen at the Lagrangian level in orienti theories,³ in fact, appears *dynamically* in the 't Hooft limit.

To see that this is indeed the case let us turn to a “refined” proof of planar equivalence presented in [29]. The analysis is based on N_f expansion in the given “background” gluon field, with the subsequent integration over the gluon field. Figure 4 displays one-fermion loop in planar geometry (i.e. on a sphere). The gluon fields “inside” and “outside” the loop do not communicate with each other at $N \rightarrow \infty$. This is indicated by distinct shadings. Averaging over the gluon field inside the loop is independent of averaging outside. This means that in calculating this contribution we can introduce two distinct gluons, two independent $SU(N)$ ’s. Each has its Z_N center. However, only one of them survives due to the fact that the propagating fermion has two fundamental indices upstairs. A typical multiloop contribution (five fermion loops) is shown in Fig. 5. Here we have six distinct $SU(N)$ gluons, with five constraints on six Z_N center groups. Again, one Z_N center survives. Needless to say, this symmetry disappears at $1/N$ level.

Thus, we do have a Z_N center-group symmetry in orienti-S/AS! Conceptually, this is a non-trivial statement, at the same time being a trivial consequence of planar equivalence. After all (reduced) SUSY is also not obvious apriori in orienti theories. This observation invalidates some statements in the literature; in particular, it restores “equal-rights” status for even and odd values of N .

³For even N there is, of course, an obvious Z_2 center group, which is no match to Z_N

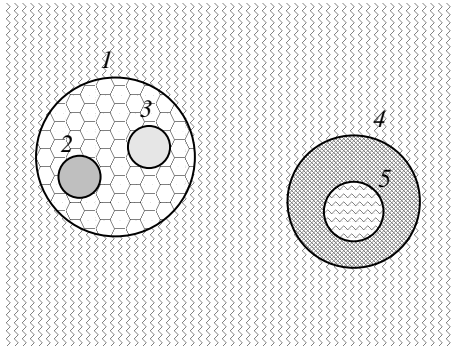


Figure 5: An example of fermion multiloops in the gluon field background at $N \rightarrow \infty$.

6 D Branes in Field Theory

In 1996 Dvali and I reanalyzed [7] supersymmetric gluodynamics, found an anomalous $(1, 0)$ central charge in superalgebra, not seen at the classical level, and argued that this central charge will be saturated by domain walls interpolating between vacua with distinct values of the order parameter, the gluino condensate $\langle \lambda\lambda \rangle$, labeling N distinct vacua of the theory. An exact relation expressing the wall tension in terms of the gluino condensate was obtained. Elementary walls interpolate between vacua n and $n + 1$, while k -walls interpolate between n and $n + k$.

In 1997 Witten interpreted [37] the above BPS walls as analogs of D-branes. This is because their tension scales as $N \sim 1/g_s$ rather than $1/g_s^2$ typical of solitonic objects (here g_s is the string constant). Many promising consequences ensued. One of them was the Acharya–Vafa derivation of the wall world-volume theory [38]. Using a wrapped D -brane picture and certain dualities they identified the k -wall world-volume theory as 1+2 dimensional $U(k)$ gauge theory with the field content of $\mathcal{N} = 2$ and the Chern-Simons term at level N breaking $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. Later Armoni and Hollowood exploited this set-up to calculate the wall-wall binding energy [39].

In 2002 Yung and I considered $\mathcal{N} = 2$ model, weakly coupled in the bulk (and, thus, fully controllable), which supports both BPS walls and BPS flux tubes [8]. We demonstrated that a gauge field is indeed localized on the wall; for the minimal wall this is a $U(1)$ field while for nonminimal walls the localized gauge field is non-Abelian. We also found a BPS wall-string junction related to the gauge field localization. The field-theory string does end on the BPS wall, after all! The end point of the string on the wall, after Polyakov’s dualization, becomes a source of the electric field localized on the wall. In 2005 Sakai and Tong analyzed [40] generic wall-string configurations. Following condensed matter physicists they called them *boojums*.

Among advances of the recent years I want to single out one development: the

so-called moduli matrix approach for description of BPS-saturated multiwall and multistring configurations and their junctions in $\mathcal{N} = 2$ $U(N)$ theories [41, 42] (for a review see [43]). The moduli matrix is shown to be a useful tool in exhaustive description of all BPS solutions since it provides a universal parametrization for complicated moduli spaces typical in multiwall and multistring configurations. The total moduli space of multiwalls is demonstrated to be given by the complex Grassmann manifold $SU(N_f)/[SU(N) \times SU(N_f - N) \times U(1)]$. It can be decomposed into various topological sectors corresponding to boundary conditions specified by particular vacua. Charges characterizing composite solitons give a negative contribution in Abelian theories and can contribute either positively or negatively) in non-Abelian theories. Various applications of the moduli matrix approach were worked out, for instance, interaction rules of monopoles, vortices, and walls were derived. Among other applications, as a test, a detailed analysis of the moduli space of axially symmetric 2-strings in $U(2)$ gauge theory with two flavors was carried out [44]. The moduli space in this case is $CP(2)/Z_2$, a manifold containing an A_1 -type (Z_2) orbifold singularity. This limiting case coincides with the answer obtained in the same problem previously [45] by a totally different method.

7 Advanced perturbative calculations with gluons and quarks

In gauge theories obtaining high orders in the perturbative expansion (multiparton scattering amplitudes at tree level and with loops) is an immense technical challenge. Due to the gauge nature of interactions, the final expressions for such amplitudes are orders of magnitude simpler than intermediate expressions.

In the 1990's Bern, Dixon and Kosower pioneered applying string methods to obtain loop amplitudes in supersymmetric theories. The observed simplicity of these results (generalizing the elegant structure of the Parke-Taylor amplitude [46]) led to an even more powerful approach based on unitarity. Their work resulted in an advanced helicity formalism exhibiting a feature of the amplitudes, not apparent from the Feynman rules, an astonishing simplicity. In 2003 Witten uncovered [47] a hidden and elegant mathematical structure in terms of algebraic curves in terms of twistor variables in gluon scattering amplitudes: he argued that the unexpected simplicity could be understood in terms of twistor string theory.

This observation created a diverse and thriving community of theorists advancing perturbative calculations at tree level and beyond, as it became clear that loop diagrams in gauge theories have their own hidden symmetry structure. Most of these results do not directly rely on twistors and twistor string theory, except for some crucial inspiration. So far, there is no good name for this subject. Marcus Spradlin noted that an unusually large fraction of contributors' names start with

the letter B.⁴ Therefore, perhaps, we should call it B theory, with B standing for beautiful, much in the same way as M in M theory stands for magic. I could mention a third reason for “ B theory”: Witten linked the scattering amplitudes to a topological string known as the “ B model.”

B theory revived, at a new level, many methods of the pre-QCD era, when S-matrix ideas ruled the world. For instance, in a powerful paper due to Britto, Cachazo, Feng and Witten (BCFW) [48], tree-level on-shell amplitudes were shown in a very simple and general way to obey recursion relations. Their proof was based only on Cauchy’s theorem and general (factorization) properties of tree-level scattering! The BCFW recursion relations gave us a way to calculate scattering amplitudes without using any gauge fixing or unphysical intermediate states.

Although the ultimate goal of the B theory is calculating QCD amplitudes, the concept design of various ideas and methods is carried out in supersymmetric theories, which provide an excellent testing ground. Looking at super-Yang–Mills offers a lot of insight into how one can deal with the problems in QCD.

Of all supersymmetric theories probably the most remarkable is $\mathcal{N} = 4$ Yang–Mills. Its special status is due to the fact that (a) it is conformal, and (b) in the planar strong coupling limit it is dual to string theory on $\text{AdS}_5 \times \text{S}^5$.

In 2005 Bern, Dixon and Smirnov calculated in this theory 2 gluons $\rightarrow 2$ gluons amplitude up to three loops [49]. Based on this and earlier results with Anastasiou and Kosower [50] they suggested an ansatz for the maximally helicity violating n -point amplitudes to all loop orders in perturbation theory in the planar limit. For 2 gluons $\rightarrow 2$ gluons amplitude the Bern-Dixon-Smirnov conjecture takes the form

$$\mathcal{A}(2 \text{ gluons} \rightarrow 2 \text{ gluons}) = \mathcal{A}(2 \text{ gluons} \rightarrow 2 \text{ gluons})_{\text{tree}} \times \exp \left[(\text{IR divergent}) + \frac{f(\lambda)}{8} \left(\ln \frac{s}{t} \right)^2 + \text{const.} \right] \quad (9)$$

where λ is the ’t Hooft coupling and the function $f(\lambda)$ is directly related with the cusp anomalous dimension.

Recently there was an elegant development in this issue due to Alday and Maldacena [51]. In a tour-de-force work they performed the *strong* coupling computation by using the gauge theory/gravity duality that relates $\mathcal{N} = 4$ Yang–Mills to string theory on $\text{AdS}_5 \times \text{S}^5$. They found that the leading order result at large values of the ’t Hooft coupling λ is given by a single classical string configuration. The classical string solution depends on the momenta k_i^μ of the final and initial gluons. The

⁴E.g. Badger, Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, ... (Of course, one should not forget about Cachazo, Dixon, Feng, Forde, Khoze, Kosower, Roiban, Spradlin, Svrček, Travaglini, Vaman, Volovich, ...). This reminds me of a joke of a proof given by a physicist that almost all numbers are prime: one is prime, two is prime, three is prime, five is prime, while four is an exception just supporting the general rule.

Alday-Maldacena strong coupling result perfectly matches Eq. (9)! It should be stressed that it is an AdS/CFT match for a rather non-trivial dynamical quantity.

8 $\mathcal{N} = (2, 0)$ sigma model (heterotic flux tube)

Now I want to discuss a stimulating recent development due to Edalati and Tong: heterotic flux tubes in $\mathcal{N} = 1$ theories. As was mentioned, non-Abelian flux tubes were first discovered in $\mathcal{N} = 2$ SUSY Yang–Mills with the appropriately chosen matter sector (8 supercharges). The flux tube solutions are 1/2 BPS. Hence, the effective low-energy theory of moduli fields on the string worldsheet must have four supercharges.

In this problem there are two classes of bosonic moduli which split from each other: orientational moduli and two translational moduli. The orientational moduli form $\text{CP}(N - 1)$ model, while the translational ones do not interact. Then the requirement of four supercharges in two dimensions unambiguously leads us to $\mathcal{N} = (2, 2)$ $\text{CP}(N - 1)$ model on the string world sheet. (Of course, in addition, there is the $\mathcal{N} = (2, 2)$ free theory of translational/supertranslational moduli, which is dynamically trivial and thus uninteresting.)

An intriguing question arises when one deforms the bulk theory to break $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. If this is done in a judicious way, e.g. through a superpotential $\mathcal{W}(\mathcal{A})$ for the adjoint fields,⁵ 1/2 BPS flux tube solutions stay essentially intact. Moreover, the number of the boson and fermion zero modes, which become moduli fields on the string worldsheet, does not change either. However, the bulk theory has now only four supercharges. According to the standard logic this would imply two supercharges in the string worldsheet theory.

If superiorientational and supertranslational modes are decoupled, supersymmetrization of the orientational and translational modes occurs separately. The orientational modes form $\text{CP}(N - 1)$ model. As well-known, requiring $\mathcal{N} = 1$ SUSY in $\text{CP}(N - 1)$ automatically leads to a nonchiral model with extended supersymmetry, $\mathcal{N} = (2, 2)$. This was the line of reasoning Yung and I followed in 2005 [52] in arguing that non-Abelian strings obtained in $\mathcal{N} = 1$ bulk theories with $\mathcal{W}(\mathcal{A}) \propto \mathcal{A}^2$ have enhanced supersymmetry.

This would be certainly true if the worldsheet theory was just $\text{CP}(N - 1)$ sigma model. In fact, it is $C \times \text{CP}(N - 1)$ sigma model. Edalati and Tong noted [53] that the latter does have a generalization with two supercharges due to the fact that superiorientational and supertranslational modes gets entangled. In two dimensions there are two distinct superalgebras with two supercharges: the nonchiral $(1, 1)$ algebra and the chiral $\mathcal{N} = (2, 0)$ algebra. It is the latter which was shown [53]

⁵The simplest choice is $\mathcal{W}(\mathcal{A}) \propto \mathcal{A}^2$. Inclusion of higher powers of \mathcal{A} is possible.

to be relevant. As a result, the right- and left-moving fermions acquire different interactions, and the flux tube becomes heterotic!

In essence, the deformation of the $CP(N-1)$ model found by Edalati and Tong reduces to a four-fermion interaction coupling left-handed superorientational fields $\Psi_1^{\dagger\bar{j}}$ and Ψ_1^i with the left-handed fermion fields η_1, η_1^\dagger originating from the would-be supertranslational modes (the two modes corresponding to two supercharges that are lost in the transition from $\mathcal{N} = 2$ to $\mathcal{N} = 1$). It is of the type $R_{i\bar{j}} \Psi_1^i \Psi_1^{\dagger\bar{j}} \eta_1 \eta_1^\dagger$. Direct derivation of such terms from the bulk theory is quite tricky. Instead, Edalati and Tong considered a class of superpotentials $\mathcal{W}(\mathcal{A})$ in the bulk theory, derived the corresponding bosonic sector of the worldsheet theory and then reconstructed the fermionic sector using $\mathcal{N} = (2, 0)$. As an independent check they showed that various symmetries of the bosonic sector of the worldsheet theory match symmetries of the bulk theory.

Direct derivation of the fermionic sector of string worldsheet theories for various $\mathcal{N} = 1$ bulk theories supporting non-Abelian 1/2 BPS flux tubes remains an open question.

Very promising developments are expected to follow [54].

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